

The Tethered Cow (or Goat on a Rope) Problem

Students are first introduced to the goat problem in geometry when they are asked to determine the grazing area of an animal that is tied to a stake in the middle of an open yard. In precalculus the problem is expanded to include a fenced in area with a building; while in trigonometry, the problem is used as an application for area of triangles and the Laws of Sine and Cosine. Each of these problems can be examined with the Geometer's Sketchpad by constructing sectors of circles, rectangles, and triangles.

Sample problems (of increasing difficulty):

Directions:

Use the Geometer's Sketchpad to illustrate and solve each problem:

1. If a goat is tied with a 10-foot rope to a stake in an open field, how much grazing area does he have?
2. The back of a 20 foot by 40 foot barn adjoins an 100 foot fence. If a goat is tied with a 16 foot rope to the fence post that joins the barn, how much grazing area in the barnyard does the goat have?
3. A dog is tied with a 6 foot rope to a corner of a building that is 15 feet by 15 feet. How much running area does the dog have?
4. A cow is tethered to a post alongside a barn 10 meters wide and 30 meters long. If the rope is 10 meters from a corner of the barn and if the barn is 30 meters long, find the cow's total grazing area to the nearest square meter. (from *Richard G. Brown's Advanced Mathematics--Houghton Mifflin*)

5. A cow is tethered to one corner of a square barn, 10 feet by 10 feet, with a rope 100 feet long. What is the maximum grazing area of the cow?

(This problem is from *Sullivan and Sullivan's Precalculus--Prentice Hall*, but it was suggested by Professor Teddy Koukounas of SUNY at Old Westbury, who learned of it from an old farmer in Virginia. The Law of Sines will be needed to find a small triangular portion of this area.)

6. Let a circular field of unit radius be fenced in, and tie a goat to a point on the interior of the fence with a chain of length r . What length of chain must be ordered to allow the goat to graze exactly one half the area of the field?

(<http://mathworld.wolfram.com/GoatProblem.html>)

ANSWERS

1. 314.159 square feet
2. 201.062 square feet
3. 84.823 square feet
4. 1885 square meters
5. 1644.9 square feet
6. approximately 1.159 units

WEB SITES FOR GEOMETER'S SKETCHPAD APPLICATIONS

1. Cynthia Lanius has directions for rotating, translating, reflecting, and animating pinwheels. <http://math.rice.edu/~lanius/misc/hando.html>
2. Among many sketchpad activities, Jim King uses the sketchpad to model a **Ferris Wheel**. <http://mathforum.org/sketchpad/sketchpad.html>
3. A **Teacher's Teaching Technology** geometer's sketchpad class posted their lesson plans at <http://ttt.ssd.k12.wa.us/projects/geo/>

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Geometer's Sketchpad
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West Virginia Instructional Goals and Objectives:

Algebra/Geometry Preparation: 10, 11, 23

Geometry: 16, 21

Algebra II: 16, 19, 27

Trigonometry: 4, 5, 8, 9, 11, 16, 17, 18, 19

Precalculus: 14, 23