Applications of Linear Equations Unit-Allen Meck

Objectives:

M.1.HS.RBQ.1	***use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret the scale and the origin in graphs and data displays***
M.1.HS.RBQ.6	***create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales***
M.1.HS.LER.1	***understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.***
M.1.HS.LER.18	***interpret the parameters in a linear or exponential function in terms of a context***
M.1.HS.RWE2	***solve linear equations and inequalities in one variable***

Overview:

Day 1	Activity: Linear Circles
Day 2	Solving 1-Step Equations
Day 3	Activity: Writing Linear Equations
Day 4/5	Solving 2-Step Equations
Day 6	Activity: Working with Linear Equations
Day 7/8	Solving Multi-Step Equations
Day 9	Activity: Buggy Lab
Day 10/11	Solving Equations with variables on both sides
Day 12/13	Activity: The Rematch
Day 14	Activity: Figuring with Flying
Day 15/16	Activity: Kinesthetic Graphing
Day 17	Solving Equations with Infinite Solutions or No Solution
Day18/19	Activity: Linear Cup Stacking
Day 20	Activity: The Phone Wars
Day 21	Activity: Balloon Festival
Day 22/23	Unit Review

Differentiated Instruction Chart

	Content	Process	Product
Readiness	As we translate verbal situations to algebraic descriptions, students of different readiness can be called upon to: a) identify specific values of the dependent variable, or b) to describe the pattern in general terms, or c) to give the algebraic description.	As each type of linear equation is covered, some students will demonstrate greater proficiency. When the class is working in pairs, they can be grouped to allow stronger students to assist those still developing their skills.	Writing their own linear equations will allow students to perform at their individual ability levels. It will also allow them to demonstrate progress as their problems increase in sophistication.
Interest	By varying the context of the problem sets and the types of activities, I hope to touch on at least one area of interest for each student.	The various data collections allow students to choose to be data recorders, timers, measurers, or kinesthetic participants.	Writing their own verbal expressions to translate to algebraic equations allows each student to tailor the problems to areas of their choice.
Learning Profile	While ALL students must demonstrate minimum competencies, some will be able to be challenged with individual extension questions.	The variety of activities addresses those who are kinesthetic (measuring, or running), auditory (hearing others share their descriptions), and visual (creating and interpreting graphs)	Students will work in groups to complete the final project. Some will create graphs, others write the story line, and others create equations and solutions.

Student Indicators

1) Students will have several quizzes during this unit. This will help assess their mastery.
2) Students will be completing daily warm up activities which will be collected and evaluated periodically.
3) Students will be discussing their answers to the various activities both within groups and with the teacher.
4) Students will be creating their own linear equations and problem scenarios. These will provide another method for assessing their understanding.

Lesson Highlights:

Day 1 Linear Circles Students are given centimeter measuring tapes and a large variety of cylindrical objects. (Preferably at least ten objects ranging from the size of a film canister to a classroom trash can) They work in pairs to measure the object's height, diameter and circumference. Data can be graphed by hand or with a graphing utility. Students should be led to determine reasonable values for domain, range and scaling. Plotting diameter vs. height yields a completely random graph, while plotting circumference vs. diameter yields a very nicely linear graph with a slope of pi. I do not allow students to use regression, but rather help them "discover" why the vertical intercept must be zero and then find the slope of a line of best fit through the origin. For many students this is their first tactile experience with this important constant. It also presents the opportunity to discuss why the slope has no units even though both axes do.

Day 2 Solving 1-Step Equations Students are reminded of the grade-school problems where they must fill in the box. Then we discuss x filling the role of the box. I start with simple problems which students can easily solve by inspection. Move to problems involving integers, decimals and fractions to force the development of solving strategies. Emphasize the inverse operations being used and checking solutions by substitution.

Day 3 Writing Linear Equations A sample worksheet is included. This day focuses on translating verbal descriptions of a linear scenario into algebraic equations. Much time is devoted to the idea of slope representing how quickly a quantity is changing and y-intercept representing an initial value or starting point. Beyond the worksheet, students are asked to write their own descriptions of problems, exchange with another student and then translate into algebraic form. I circulate around during this time, having students explain their thought processes and asking probing questions like "What could make your slope negative?" or "Is it possible for this scenario to have a negative intercept?"

Day 4/5 Solving 2-Step Equations These days are spent clarifying the need to use inverse operations in the reverse order of PEMDAS. I like to use the Rubik's Cube to illustrate this concept. By showing that to undo a series of moves, we must not only do the opposite move, but also in the reverse order students who have played with a cube grasp the idea. We emphasize that as soon as the first operation is performed, we are back to a simple equation as previously solved. In particular, students struggle with equations involving division and fractions, so I cover these in depth.

Day 6 Working with Linear Equations The exercises on this worksheet, and the additional problems worked in class, are less step-by-step than the ones completed previously. Students have to identify the slope, and intercept in order to write the appropriate equation, but they are not prompted for these values. Then the equations are used to solve problems and address important issues. We consider questions that can be answered by simple substitution, some that require substituting and then solving, and others that begin to develop the idea of limitations on the domain and range of linear functions.

Day 7/8 Solving Multi-Step Equations By this point, students have developed a familiarity with the basic steps of equation solving, so we are simply extending strategies to include collecting like terms, distributing through parentheses, or dealing with division as a grouping symbol.

Day 9 Buggy Lab In this activity, students use a battery powered dune buggy to collect distance vs. time data. I have also used radio-controlled cars or even rolling marbles over short distances. They key is to have a data set that clearly indicates a linear relationship and thus constant velocity. I usually use floor tiles as my distance unit, but inches, meters or any other convenient measurement would work fine. Cell phones with lap timing capability make data collection quick so that we can analyze it during the same class. Like the Linear Circles Activity, this data will produce a linear pattern with very little error. For this reason, it is a nice set to be graphed by hand (though this would likely extend the activity to a second day). Be sure to have students address what a non-zero intercept would mean and explore with them why slope represents average velocity. If you have several buggies available, you can put partially dead batteries in some (or run them uphill) to change their velocity and thus see the effect on the graphed data.

Day 10/11 Solving Equations with variables on both sides The emphasis of this time is on making sure students are seeing the difference between collecting like terms on the same side of an equation vs. moving terms from one side to the other. Some students struggle with trying to put variable terms together with constants. This is an excellent opportunity for pairs-check type of activities so that no one gets too far down a wrong path without correction. I like to have some students working at their seats while others put the sayme problems on the board and explain them. This is also when I introduce the idea of two expressions (which could each represent a linear modeling) being set equal to each other to find a common value.

Day 12/13 The Rematch This was the first linear equation activity I developed after getting the seed of the idea from a conference I attended. I emphasize the slope – velocity and the head start – initial value relationships. I require students to use the equations to solve each part (even though some can do so in their heads) because I want them to practice using equations in a non-threatening situation where they have an intuitive feel for the correctness of their answers. We graph this data by hand. Depending on the level of the group, this activity may take more or less time. Question q generally requires a good deal of prompting.

Day 14 Figuring with Flying This activity is very similar to The Rematch. Depending on the mastery demonstrated in that task, students may be able to complete this activity in small groups with only modest amounts of teacher input. However, some classes may still need a more structured coverage, working through each question together or working a single question independently and then discussing it.

Day 15/16 Kinesthetic Graphing Despite my initial hesitation that students might find this too immature, most of them love it. We go out to the track or parking lot and mark off increments in meters, yards or feet. Then we time students as they walk, jog, run, hop, skip, crabwalk, backwards walk, sidestep, three-legged walk, etc. I try to have students start before the beginning mark so that they are already at a steady pace when we begin timing. The data sets are not always perfectly linear, but certainly enough that they illustrate the point. This is a particular favorite of my kinesthetic students.

Day 17 Solving Equations with Infinite Solutions or No Solution The focus of this day is considering what it means to solve an equation. Specifically, we are identifying a value for the variable that makes the original statement true. So we explore what happens when two linear equations cross at a single point, are parallel, or completely overlap. I also try to approach this from a "common sense" perspective. For example I emphasize that 5x + 2 can NOT ever be the same as 5x + 3, no matter what value we assign to x. Meanwhile, it will ALWAYS be true that 5x + 2 = 5x + 2. Finally, I try to make clear to students that arriving at a statement like 2 = 3 or 5 = 5 is NOT the place to stop. They need to draw a conclusion.

Day 18/19 Linear Cup Stacking Students measure the height of stacks of nested styrofoam cups. Ideally, there should be at least 10 stacks with a different number of cups in each. (Note that the cups must all be the same type.) Students then determine a linear equation to fit the data. The resulting graph is different from the other activities in that it has a non-zero intercept. Explaining why this is the case presents a challenge for the students, as does the question asking about an inverted stacking pattern.

Day 20 The Phone Wars . I developed this activity several years ago, in response to the constant battle that MCI, Sprint, ATT and other carriers were waging for my business. It presents what was once a very common scenario. Now it requires a little explanation since so many people have "unlimited" plans. The questions give students the chance to consider the relative importance of the slope and the intercept in a practical sense. Although, the plans are no longer current, I still believe this activity has much to offer in terms of the mathematical insights that students can gain.

Day 21 Balloon Festival This activity is very similar to Figuring with Flying and could be left out if you choose. Alternately, it could be used later as a review of previous learning.

Day 22/23 Unit Review This is another place where DI can take place. I place students in pairs and have them create their own scenarios and equations that match. They also create linear equations of each type and show fully worked out solutions which they must explain to me.

Activities Masters Follow:

Writing Linear Equations **Name ___________________________________**

1. Todd earns $5/hour at his job.
   a. What is the rate of change in this situation?

   b. What is the initial value in this situation?

   c. Use P (pay) and h (hours) to write an equation modeling his earnings.

2. Water is drained from a tank at 5 gal/min. It starts out holding 200 gallons of water.
   a. What is the slope in this situation?

   b. What is the y-intercept in this situation?

   c. Use V (volume) and m (minutes) to write an equation modeling the water in the tank.

3. You are downloading a 600K file. Your browser is running at 20K/sec.
   a. What is the rate of change in this situation?

   b. What is the initial value in this situation?

   c. Use R (remaining) and s (seconds) to write an equation modeling the amount still to load.

4. A taxi charges a $4 entrance fee plus $0.85/mile.
   a. What is the slope in this situation?

   b. What is the y-intercept in this situation?

   c. Use F (fare) and m (miles) to write an equation modeling your cost.

5. A plane flying at 5000 feet and begins ascending at 300 ft/min.
   a. What is the rate of change in this situation?

   b. What is the initial value in this situation?

   c. Use A (altitude) and m (minutes) to write an equation modeling the planes height.

6. The US birth rate is 6850 people/day. There are currently 302,652 people in the country.
   a. What is the slope in this situation?

   b. What is the y-intercept in this situation?

   c. Use P (population) and d (days) to write an equation modeling the future population.

7. Joe is cutting a 10,000 ft^2 lawn. He can cut 200 ft^2/min.
   a. What is the rate of change in this situation?

   b. What is the initial value in this situation?

   c. Use A (area) and m (minutes) to write an equation modeling the amount remaining.

Working with Linear Equations **Name ____________________________________**

1. Todd earns $1.75 for each bushel of peaches picked at his job. Use E (earnings) and b (bushels) to write an equation modeling this situation.

2. A propane tank initially contains 200 lbs of gas. It is being filled at 5 lbs/min. Use W (weight) and m (minutes) to write an equation modeling this situation.

3. Kayla has a "sugar daddy" who deposits $5000 in a bank account for her. Use B (balance) and d (days) to write an equation modeling this situation if she withdraws $125/day.

4. A line is being painted on the road at 1.5 ft/s. If the line is initially 200 ft long, model this situation with an equation using L (length) and s (seconds).

5. The temperature T of a room is originally 65 degrees and is increasing at 2 degrees/hour. Model this situation with an equation using T (temperature) and h (hours).

6. A boat is 100 yards away from a dock and being pulled in by a winch at 4 yards/min. Use D (distance) and m (minutes) to write an equation modeling this situation.

7. Charlie deposits $250/week into his bank account. His account originally had $1200 in it. Model this situation with an equation using B (balance) and w (weeks).

8. Refer to #1. How many bushels must be picked to earn $56?

9. Refer to #2. Why might this equation not be valid at m = 1000?

10. Refer to #3. What is the balance on day 7?

11. Refer to #4. When is the line 500 ft long?

12. Refer to #5. In 8 hours, what is the room temperature?

13. Refer to #6. When will the boat reach the dock?

Refer to #7. How long will it take for the balance to reach $7200?

The Rematch

Tim Turtle and Bob Bunny have both been training for their rematch. Tim has been doing wind sprints and can now maintain a blazing speed of **6 inches/sec**! Bob has worked on overcoming his napping tendencies and can now consistently run at **4 feet/sec**. The day of the rematch has finally arrived. Conceding that he can not possibly match Bob's speed, Tim requests a head start. Bob agrees that he will start at a **big rock** and Timothy can start **1000 feet** ahead at a **fallen log**. A small shallow **stream** crosses the course **600 feet** in front of the big rock. A **dirt road** crosses the course **240 feet** in front of the fallen log. The race is to be a quarter mile. ***(1 mile = 5280 ft)***

a) On the back of this page (not on graph paper) sketch a picture showing the rock, the stream, the log, the road, and the location of the finish line. Show the initial position of Bob and Tim. The drawing does not need to be to scale, but it must include the distances to each point.

b) Using $\mathbf{d_B}$ to represent distance, and **t** to represent the time since the race began, write an equation that expresses the distance Bob has traveled from the rock in terms of the time.

c) Using $\mathbf{d_T}$ to represent distance, and **t** to represent the time since the race began, write an equation that expresses the distance Tim has traveled from the rock in terms of the time. *Remember that his speed must be changed into feet/sec.*

Use the equation from part b) to answer parts d-f. Show your work.

d) At what time does Bob cross the stream?

e) At what time does Bob cross the road?

f) At what time does Bob finish the race?

Use the equation from part c) to answer parts g-i. Show your work.

g) At what time does Tim cross the stream?

h) At what time does Tim cross the road?

i) At what time does Tim finish the race?

j) Mark a set of axes on your graph paper. The __________ depends on the __________. Therefore, the

___________ will be on the vertical axis and ___________ will go on the horizontal axis.
Label what is on each axis (time or distance) and what units it is measured in (feet or seconds).

k) The longest time to finish the race is ____________. In order to fill most of the paper, how much should each block on the horizontal axis be worth?____________ Mark this axis accordingly.

l) The total length of the race was a quarter mile which is ____________ ft. In order to fill most of the paper, how much should each block on the vertical axis be worth?__________ Mark this axis accordingly.

m) Now plot the starting point and the three distance/time pairs which you found for Bob in parts d-f on your graph. Connect these points. They should all be on one line.

n) Now plot the starting point and the three distance/time pairs which you found for Tim in parts g-i on your graph. Connect these points. They should all be on one line.

o) On your graph, draw dashed horizontal lines to indicate the positions of the rock, the log, the stream, the road, and the finish line.

p) By how much time does the winner win the race? (Show how you get your answer)

q) By how much distance does the winner win the race? (Show how you get your answer)

Figuring with Flying

Purple Planes, Amber Airlines, and Fushia Flights have all taken to the skies!

Purple #305 is cruising at 10,000 feet when it receives instructions to ascend at 50 ft/s.
At the same instant, Amber # 444 receives clearance to take off and climb at 80 ft/s.
Also at the same instant, Fushia #007is directed to begin descending from 20,000 feet at 25 ft/s.

1. Write an equation using P (Purple's height) and t (time in seconds) to model the altitude of #305.

2. Write an equation using A (Amber's height) and t (time in seconds) to model the altitude of #444.

3. Write an equation using F (Fushia's height) and t (time in seconds) to model the altitude of #007.

4. What does the slope (including the sign) of each of the above equations tell us about what is going on?

5. What does the y-intercept of each of the above equations tell us about what is going on?

6. Set up and solve an equation for each plane to determine when it reaches 15,000 feet.

   <u>Purple</u> <u>Amber</u> <u>Fushia</u>

7. Set up and solve an equation to determine when Purple #305 and Fushia #007 are at the same height.

8. Set up and solve an equation to determine when Fushia #007 will land.

9. Set up and solve an equation to determine when Purple #305 and Amber #444 are at the same height.

10. Set up and solve an equation to determine when Amber #444 reaches 30,000 feet.

11. Set up and solve an equation to determine when Amber #444 and Fushia #007 are at the same height.

12. Fifteen seconds after receiving their instructions, what is the altitude of each plane?

Purple	Amber	Fushia

13. If we graphed this data, the ______________________ should be placed on the vertical axis and the ____________________ would go on the horizontal axis.

14. The maximum altitude discussed is 30,000 feet. If our altitude axis had 50 blocks, show how you would determine the value of each block.

15. Suppose we want our time axis to go to 1000s. If our time axis had 35 blocks, show how you would determine the value of each block. Explain why you "round" the way you did.

16. By looking at the graph, how could you determine which plane was changing altitude fastest?

17. What would be represented by the intersection of two lines on this graph?

18. If Level Louie flew his ultralite at 5000 feet during this whole time, what would his graph look like?

Measuring the Height of Nested Styrofoam Cups

Will the height of a stack of nested cups be related to the number of cups? If yes, describe how you think they will be related. If no, explain why not.

Measure (nearest 0.1 cm) the heights of various stacks of nested cups.

Number of nested cups	Stack height (cm)

What is the dependent variable? What is the independent variable?

Enter your data into lists on the calculator.

Now set the viewing window for your graph. Explain why you will not need any negative values for either axis.

Record your window here: x-min: ___ x-max: ___ y-min:___ y-max: ___

Graph the data. Do the data seem to be related in the way you expected?
Does the pattern seem to be linear or non-linear? How can you tell?

Now use the calculator to determine the equation of best-fit:

a) by trial and error ___________
b) by a regression calculation __________

Record your preferred equation in terms of h (height) and n (number of cups).

What does the slope represent in terms of the variables investigated?

What are the units of the slope?

What does the y-intercept represent in terms of the variables investigated?

What are the units of the y-intercept?

Now let's use the equation to make some predictions. What is the height of 150 nested Styrofoam cups? Explain how you arrived at your answer.

How many cups are required to obtain a height that <u>exceeds</u> 55 cm? Explain the mathematics used to determine this number.

Suppose we used cups that had a taller rim (but the same base). How would this change the graph and the equation?

Suppose we used cups that had a shorter base (but the same rim). How would this change the graph and the equation?

Based on your equation, what are the numerical values of the -

height of the rim: _____ height of the base: _____

For the cups used, do the values above seem realistic?

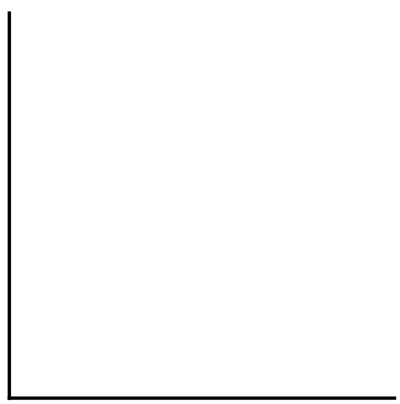
Suppose we have to pack stacks of cups into cardboard boxes for shipping. How many cups could be in a stack if it must fit into a 61 cm long box? …into a 100 cm long box?

Explain why you rounded the way you did in the previous question.

$H = 0.8n + 5.1$ is the model in inches for a different cup. What is the equation for this model in cm? (1 inch = 2.54 cm)

Using the rim & base information that you determined from your data, what equation would model stacking of the cups with inverting every other cup?

Sketch a graph for the type of stacking described above.



The Phone Wars **Name____________________________**

The long distance battle is on! Everyone wants your phone business. You are choosing from the following offers.

Plan A:	15 cents/min
Plan B:	5 cents/min with a $10 monthly fee
Plan C:	10 cents/min with a $4 monthly fee
Plan D:	7 cents/min with a $7 monthly fee

1. Use **A**, **B**, **C** & **D** to represent the cost of each plan and **m** to represent the minutes talked. Write equations that represents the charges and time usage for each plan.

2. Use the notation **A(100)**, **B(100)**, **C(100)**, **D(100)** and determine the cost for each plan if you talked for 100 minutes during a month. *Show your work.*

3. Use the notation **A(30)**, **B(30)**, **C(30)**, **D(30)** and determine the cost for each plan if you talked for 30 minutes during a month. *Show your work.*

4. Use the notation **A(240)**, **B(240)**, **C(240)**, **D(240)** and determine the cost for each plan if you talked for 240 minutes during a month. *Show your work.*

5. You will now create a graph of cost vs. time. There will be no negative numbers on either the cost axis or the time axis. Explain why for:

   a) the cost axis

b) the time axis

6. The ___________________ depends on the ______________________. Therefore, _________________ will go on the vertical axis and ___________________ will go on the horizontal axis.

7. Set up your scale to accommodate a maximum of 240 min of talk time. Show the method you used to arrive at your answer.

8. Set up your scale to accommodate the maximum cost of any of the plans for 240 min. Show the method you used to arrive at your answer.

9. Plot the three data points for Plan A and connect them with a line. Extend your line all the way back to the vertical axis. Repeat this for Plans B, C and D.

10. Explain what the slope of each graph means.

11. Explain what the y-intercept of each graph means.

12. Would Plan A be best for someone who talks a great deal or very little? Explain why.

13. If you talked over 4 hours/month, which plan should you choose? Explain why.

14. Set up and solve an equation to find the time at which Plan B and Plan C have the same cost.

The Balloon Festival

It's time for the big balloon festival! They're everywhere, but we'll just look at a few.

Phil Phloater is initially at 1000 ft and is climbing at 30ft/min.
At the same instant, Billowin Bob is leaving the ground and climbing at 35ft/min.
Also at the same instant, Droppin Don is descending from 2500 ft at 40ft/min.

19. Write an equation using P and t (time in min) to model Phil's altitude.

20. Write an equation using B and t (time in min) to model Bob's altitude.

21. Write an equation using D and t (time in seconds) to model Don's altitude.

22. What does the slope (including the sign) of each of the above equations tell us about what is going on?

23. What does the y-intercept of each of the above equations tell us about what is going on?

24. Set up and solve an equation for each balloon to determine when it reaches 2000 feet.

Phil	Bob	Don

25. Set up and solve an equation to determine when Phil and Bob are at the same height.

26. Set up and solve an equation to determine when Don will land.

27. Set up and solve an equation to determine when Phil and Don are at the same height.

28. Set up and solve an equation to determine when Bob reaches 3000 ft.

29. Set up and solve an equation to determine when Bob and Don are at the same height.

30. Fifteen minutes after receiving Bob takes off, what is the altitude of each balloon?

Phil	Bob	Don

31. If we graphed this data, the ______________________ should be placed on the vertical axis and the ___________________ would go on the horizontal axis.

32. We want a graph to accommodate 10,000 ft. If our altitude axis had 30 blocks, show how you would determine the value of each block.

33. Suppose we want our time axis to go to 1000 min. If our time axis had 35 blocks, show how you would determine the value of each block. Explain why you "round" the way you did.

34. By looking at the graph, how could you determine which balloon was changing altitude fastest?

35. What would be represented by the intersection of two lines on this graph?

36. Stable Stan maintains a constant altitude of 1800 ft. When does each balloon pass him?

Phil	Bob	Don

Online Resources

https://www.khanacademy.org/math/algebra/solving-linear-equations-and-inequalities

This site provides tutorials covering a variety of equation solving techniques.

http://illuminations.nctm.org/LessonDetail.aspx?ID=L782

This NCTM link provides directions for a Battleship type of linear equation practice.

http://math.hws.edu/javamath/basic_applets/SliderGraph.html

This is an applet that allows you to input a linear function and then manipulate the slope and intercept values by adjusting a slider.