ALGEBRA II

IGO Test Bank

SOLUTIONS

A2.1

1. (a) | a |

2. $\frac{1}{3} < \frac{1}{x+1} < 2\ 0$

   $2 < x + 1 < 3$
   $1 < 2x + 2 < 6$
   $-1 < 2x < 4$
   $-2 < x < 2$

3. (b + c)a

4. (d) a - bi

5. (c) -22

6. (a) 2

7. (b) x < -1

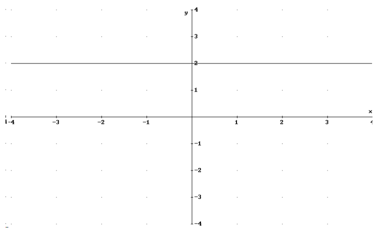
8. (b) 78

9. (c) Commutative Property of Multiplication and Commutative Property of Addition

A2.2

1. For example: $y = 2$

   a.



   b. (1,2) (2,2) (3,2)

   c. x-intercept does not exist
   y-intercept (0,2)

   d. slope: $m = 0$

2. (c) (a,d)

3. $5x - 2y = 24$

4. (a) $y = -x - 2$

   The slope of the line containing (2, -4) (-3, 1) is:

   $M = \frac{1-(-4)}{-3-2} = \frac{5}{-5} = -1.$ Using point-slope form $y - y_0 = m(x - x_0)$ we

   have $y + 4 = -1(x-2)$ Ψ $y + 4 = -x + 2$ Ψ $y = -x + (-2)$ or $y = -x - 2$.

5. (c) $-\frac{5}{2}$

6. (a) $\frac{5}{2}$

7. (c) $y = -3x + 1$

8. $m = \frac{5-4}{-2-3} = \frac{1}{-5}$

$y - 5 = \frac{-1}{5}(x + 2)$

$y - 5 = \frac{-1}{5}x - \frac{2}{5}$

$y = \frac{-1}{5}x + \frac{23}{5}$

or

$y - 4 = \frac{-1}{5}(x - 3)$

$y - 4 = \frac{-1}{5}x + \frac{3}{5}$

$y = \frac{-1}{5}x + \frac{23}{5}$

9. $3x + 4y + 5 = 6$

$4y = -3 + 1$

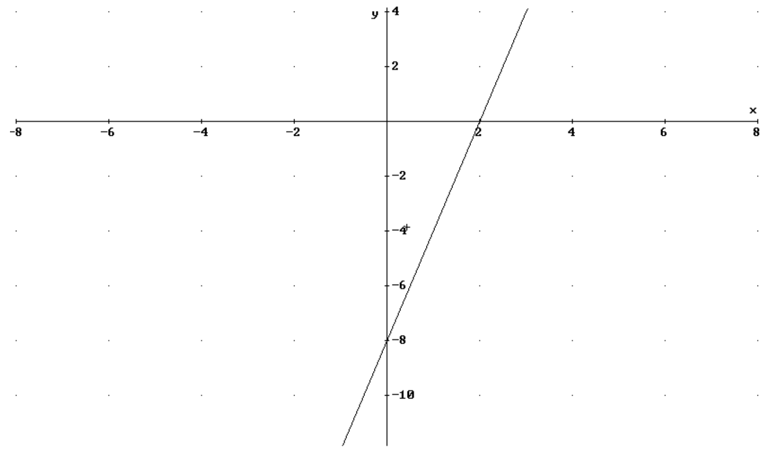
$y = \frac{-3}{4}x + \frac{1}{4}$

$m = \frac{-3}{4}$

10. (b) $y = -\frac{21}{10}x - \frac{8}{5}$

11. (a) $-3x + y = 11$

12.



2.3

1. $x(p + 1) + p + 1$
   $x (p + 1) + 1(p + 1)$
   $(p + 1) (x + 1)$

2. (b) $x^{1/2} (1 + 3x^{1/2})$

3. $3(x + 2)(x^2 - 2x + 4)(x - 1)(x^2 + x + 1)$

4. $x^2 + 2x + 4$ The factors for $a^3 - b^3$ are $(a - b) (a^2 + ab + b^2)$ therefore the factors for $x^3 - 8$ are $(x - 2) (x^2 + 2x + 4)$. An alternate procedure would be to divide $x^3 - 8$ by $x - 2$ either by standard or synthetic division.

5. (c) $(x^2 + 5)(x^4 - 5x^2 + 25)$

6. $(2x - 3)(3x - 2)$

7. $8x^3 - 27$
   $(2x - 3)(4x^2 + 6x + 9)$ since $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

8. (c) $(4x^2 + 1)(2x + 1)(2x - 1)$

9. (b) $2y(x - 3)(x^2 + 3x + 9)$

10. (d) (3m + 2n)(m + 4n)

11. (a) (x + 2)(x + 3)(x - 3)

A2.4

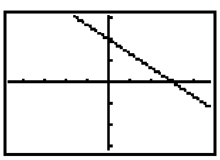
1. $-1 \leq \frac{2-3x}{4} \leq 5$

- 4 # 2 - 3x # 20
- 6 # - 3x # 18
2 ∃ x ∃ -6 or -6 # x # 2

2. If $\frac{1}{2x-1} > 0$ then 2x - 1 > 0

2x - 1 > 0
2x > 1
(c) x > 2

3.



Shade region which includes (0 , 0).

4. (d) 2x + 3y # 6

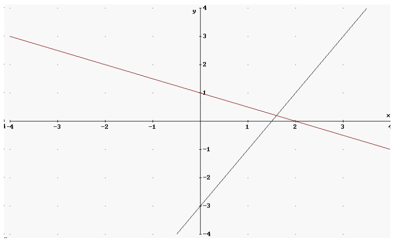
The boundary line passes through the points (0,2) and (3,0). The slope,

$m = \frac{0-2}{3-0} = \frac{-2}{3}$; the y-intercept is (0,2) the equation is $y = \frac{-2}{3}x + 2$

$3y = 3\left[-\frac{2}{3}x + 2\right] \Rightarrow 3y = -2x + 6 \Rightarrow 2x + 3y = 6.$ By testing the point

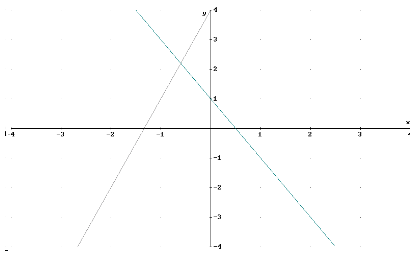
(0,0), 2(0) + 3(0) < 6, gives us the inequality 2x + 3y# 6.

5.



Shade region which includes (3 , 0).

6.



Shade region which includes (-2 , 0).

7. $5x + 7 > 2x - 9$

$3x + 7 > -9$

$3x > -16$

$x > \frac{-16}{3}$

8. [2, 4)

A2.5

1. (d) 2 - 2i

2. $\frac{4}{5} + \frac{3i}{5}$

3. (b) -2 - 16i

$(2 + 3i)(-4 - 2i) = 2(-4) + 2(-2i) + (3i)(-4) + (3i)(-2i)$ Ψ $-8 - 4i - 12i - 6i^2$ Ψ $-8 - 16i - 6(-1)$ Ψ $-8 + 6 - 16i$ Ψ $-2 - 16i$

4. $\frac{7}{5} + \frac{9}{5}i$ Solution: $= \frac{(2-3i)\,(1+i)^2}{(3-i)} * \frac{(3+i)}{(3+i)}$

$$= \frac{(2-3i)(3+i)(1+i)^2}{9-i^2}$$

$$= \frac{(9-7i)(2i)}{10} = \frac{14+18i}{10} = \frac{7+9i}{5}$$

5. (2 + 3i)(4 - 5i)

$8 - 10i + 12i - 15i^2$

8 + 2i + 15

23 + 2i

6. $(-i)^{27} = -i^{27} = -1 \text{ A } i^{24} \text{ A } i^{3} = -1 \text{ A } 1 \text{ A } -i = i$

7. (d) $-6\sqrt{6} + 0i$

8. (c) -i

9. (c) $\frac{8}{5} + \frac{1}{5}i$

A2.6

1. $(y^2+1)^{(\frac{1}{2})} + \frac{3y^2}{3(y^2+1)^{(\frac{1}{2})}} =$

$$\frac{y^2+1+y^2}{(y^2+1)^{\frac{1}{2}}} = \frac{2y^2+1}{(y^2+1)^{\frac{1}{2}}}$$

2. $\frac{y+x}{y-x}$

3. a. $12x^2y^2\sqrt{2y}$

b. $\frac{1}{81}$

4. (a) 4 $\left(\sqrt[3]{4^2}\right)\left(\frac{1}{4^{-\frac{1}{3}}}\right) = 4^{\frac{2}{3}} \bullet 4^{\frac{1}{3}} = 4^{\frac{2}{3}+\frac{1}{3}} = 4$

5. a. $x^{-3}y^4$

b. $x^{\frac{2}{3}}y^{\frac{-5}{3}}$

6. $\sqrt{\sqrt{256x^{18}}}$

$\sqrt{16x^9}$

$\sqrt{16x^8 \bullet x}$

$4x^4\sqrt{x}$

7. $\frac{x^{\frac{4}{3}}}{x^{\frac{5}{2}}} = x^{\left(\frac{4}{3}-\frac{5}{2}\right)} = x^{\left(\frac{8}{6}-\frac{15}{5}\right)} = x^{-\frac{7}{6}} = \frac{1}{x^{\frac{7}{6}}}$

8. (a) $2c^2d^3\ \sqrt[5]{d}$

9. (b) $3 \bullet 2^{\frac{1}{4}}x^{\frac{1}{2}}y^{\frac{3}{4}}z^{\frac{5}{2}}$

A2.7

1. (d) 1

2. $-2 + 2i$

3. (c) 1 $i^{16} = (i^2)^8 = (-1)^8 = 1$

4. 0 Solution: $= \frac{i^3(i^4 - 1)}{-1} = \frac{i^3(0)}{-1} = 0$

A2.8

1. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

2. (c) <

3. a. $x = \frac{5 \pm i\sqrt{39}}{4}$

   b. x = 5, 1

4. (b) two real roots

   $3x^2 - 5x - 4$; Looking at the discriminant from the quadratic formula, $b^2 - 4ac$ where a = 3, b = -5, and c = -4 we have
   $(-5)^2 - 4(3)(-4) = 25 + 48$. Since the discriminant is greater than 0 the quadratic will have **two real solutions**.

5. 0.8 and -2.5

6. $x = 0, \pm\frac{4}{3}$ Solution: $= x(9x^2 - 16) = x(3x - 4)(3x + 4)$

7. x = 8, -2 Solution: $= x^2 - 6x - 16 = (x - 8)(x + 2)$

8. $9x^2 - 27x = 0$

   $3x(3x - 9) = 0$

   $3x = 0$ or $3x - 9 = 0$

   $3x = 9$

$x = 0$ or $x = 3$

9. $4x^2 + x + 3 = 0$ $a = 4, b = 1, c = 3$

$$= \frac{-1 \pm \sqrt{1 - 48}}{8}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 + i\sqrt{47}}{8} \text{ and } \frac{-1 - i\sqrt{47}}{8}$$

10. (a) -3, 0, 3

11. (d) $\frac{3 \pm \sqrt{2}}{2}$

12. (c) $\frac{2 \pm i\sqrt{2}}{3}$

A2.9

1. $x = \frac{1}{5}$ $y = \frac{6}{5}$

2. $x = \frac{1}{10}$ $y = \frac{3}{5}$

3.

$$\begin{bmatrix} 24 & 4 & 23 \\ 13 & 11 & 12 \\ 8 & 23 & 8 \end{bmatrix}$$

4. (d) 24

$$\begin{bmatrix} 1 & -5 \\ -3 & 4 \\ 2 & -1 \end{bmatrix} x \begin{bmatrix} -4 & 2 & 0 \\ 3 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1(-4)+(-5)(3) & 1(2)+(-5)(1) & 1(0)+(-5)(-1) \\ -3(-4)+4(3) & -3(2)+4(1) & -3(0)+4(-1) \\ 2(-4)+(-1)(3) & 2(2)+(-1)(1) & 2(0)+(-1)(-1) \end{bmatrix}$$

The second row, first column is (-3)(-4) + 4(3) = 12 + 12 = **24 .**

A2.10

1. No solution. ($\sqrt{x}$ is always ∃ 0.)

2. $(x^2 + 9)^{1/2} = 5$

   $x^2 + 9 = 25$

   $x^2 = 16$

   x = ∀ 4

3. $x = \frac{1}{2}, -\frac{1}{2}, 2, -2$

4. (a) $\frac{40}{9}$ $\sqrt{x^2+1} + 4 = x - 5 \Rightarrow \sqrt{x^2+1} = x - 9 \Rightarrow (\sqrt{x^2+1})^2 =$

$$(x-9)^2 \Rightarrow x^2 + 1 = x^2 - 18x + 81 \Rightarrow 1 = -18x + 81 \Rightarrow -18x$$

$$= -80 \Rightarrow x = \frac{80}{18} \Rightarrow x = \frac{40}{9}$$

5. $t = 2$ Solution:

$$(2^3)^{t+2} = (2^4)^{2t-1}$$

$$(2)^{3t+6} = (2)^{8t-4}$$

$$3t + 6 = 8t - 4$$

$$t = 2$$

6. $x = 3, -3$ Solution: $x^2 + 1 = 10^1$ Ψ $x^2 = 9$

7. $2x^{\frac{2}{3}} = 32$

$$x^{\frac{2}{3}} = 16$$

$$(x^{\frac{2}{3}})^{\frac{3}{2}} = 16^{\frac{3}{2}}$$

$$x = 64$$

8. $x = 1$ or 0 by inspection

9. (a) no solution

10. (d) 5

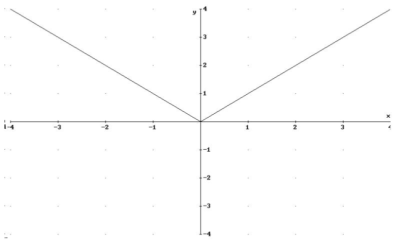
11. (c) $1, \frac{4}{9}$

12. (d) 2

A2.11

1. (0,1) (1,a)

2.



   a. (1,1) (0,0) (-1, 1)
   b. (0,0) is both x and y intercept
   c. D: | R: [0, 4)

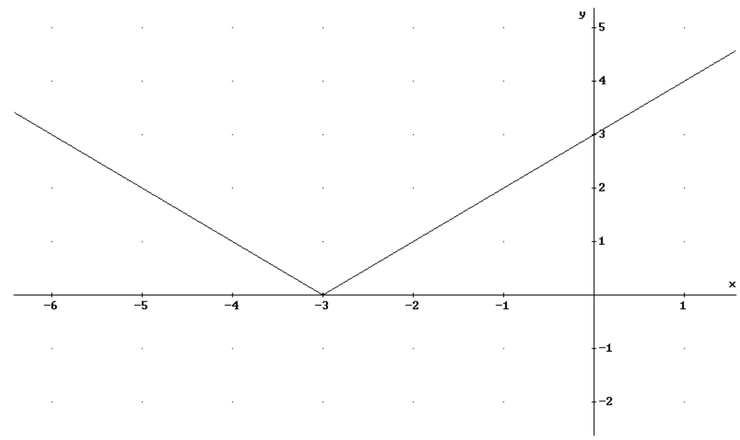
3. $y = -\frac{5}{4}x + 5$

4. $y = e^x$ Ψ

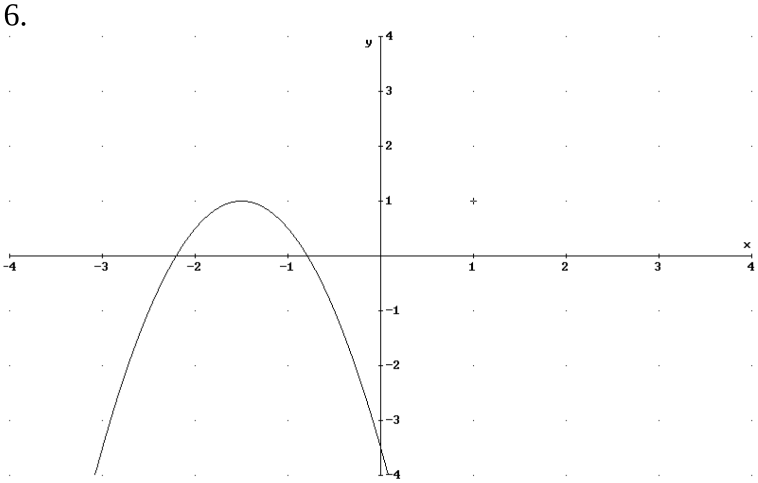
x	y
-1	1/e
0	1
1	e
2	e^2

best fits this graph.

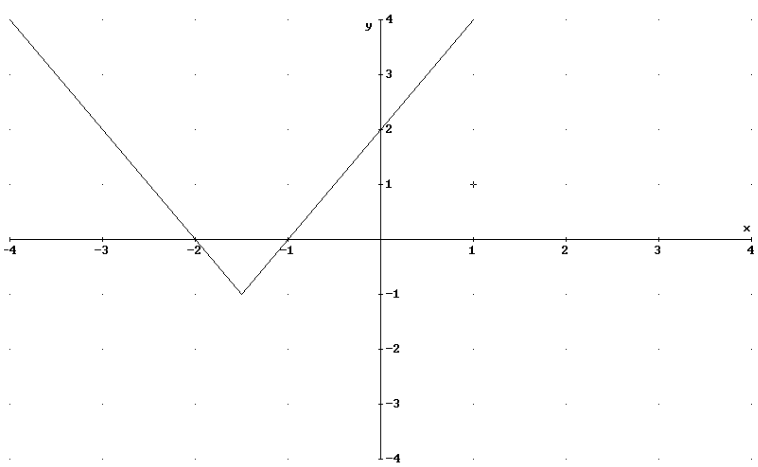
5.


y
5
4
3
2
1
-1
-2
x
-6
-5
-4
-3
-2
-1
1

6.


y
4
3
2
1
-1
-2
-3
-4
x
-4
-3
-2
-1
1
2
3
4

7.


y
4
3
2
1
-1
-2
-3
-4
x
-4
-3
-2
-1
1
2
3
4

A2.12

1. (b) (0,1) and (1,a)

2. D_f: (-4, -1) χ (-1, 4) D_{f-1}:(-4, 0) χ (0, 4) $f^{-1}(x) = \frac{2 - x}{x}$

R_f: (-4, 0) χ (0, 4) R_{f-1}: (-4, -1) χ (-1, 4)

3. (a) $x^2 + 2x + 1$

4. (b) $x^2 + 1$

5. a. $-7, \frac{1}{2}(x + 3), -2$

b. 54, 38

6. (a) $9x^2 - 30x + 25$

$f(x) = 3x - 5$ and $g(x) = x^2$ then $f(g(x)) = f(x^2) = 3x^2 - 5$

while $g(f(x)) = g(3x - 5) = (3x - 5)^2 = 9x^2 - 30x + 25$

7. Domain: $x \neq -\frac{1}{3}$

Range: $y \neq \frac{1}{3}$

Zeroes: $x = 2$

8. $4 + h$

9. D: **R**

R: $\{y \mid y \exists 0\}$

10. $f(-4) = 3(-4)^2 + 5(-4) + 2 = 48 - 20 + 2 = 30$

11. $f(g(3)) = f(2(3) - 1) = f(5) = (5)^3 = 125$

12. $y = 2^x$

x	y
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256

13. $y = 6x + 4$

$x = 6y + 4$

$x - 4 = 6y$

$$\frac{x - 4}{6} = y$$

$f(x)^{-1} = \frac{x - 4}{6}$

14. $4x^2 - 25 = 0$

$(2x + 5)(2x - 5) = 0$

$2x + 5 = 0$ or $2x - 5 = 0$

$2x = -5$ $2x = 5$

$x = \frac{-5}{2}$ or $x = \frac{5}{2}$

15. a. (a) $\frac{\sqrt{14}}{2}$

b. (d) [2, 4)

c. (b) [0, 4)

16. a. (a) $x^2 + 1 + \sqrt{x}$

b. (c) [0, 4)

c. (a) $\frac{x^2 + 1}{\sqrt{x}}$

d. (c) (0, 4)

A2.13

1. $v = (1)(x - 2)^2$
$4 = x^2 - 4x + 4$
$0 = x^2 - 4x$
$0 = x(x - 4)$ $x = 0$ or $x = 4$

The sheet should be 4 cm. H 4 cm.

2. $y = 3x^2 + 5x - 2$

a. (0, - 2) y-intercept; (- 2, 0) (1/3, 0) x-intercepts
b. (- 5/6, - 4 1/12)
c. x = - 5/6
d. D: | R: [- 4 1/2, 4)
e. opens up

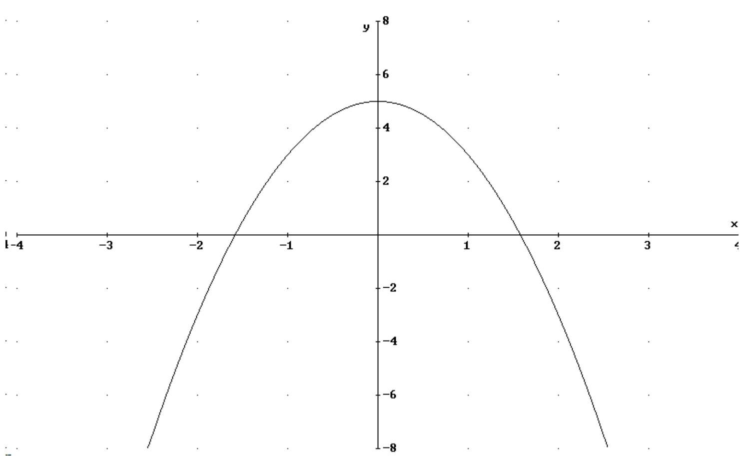
3. 64 cm^2
4. (d) x # -6 or x ∃ 1

If $x^2 + 5x - 6$ ∃ 0 then (x + 6)(x - 1) ∃ 0 . This implies that x + 6 = 0 and x - 1 = 0. So x = -6 and x = 1 are split points on the number line. All numbers x < -6 are positive in the quadratic $x^2 + 5x - 6$ while all number -6 < x < 1 are negative , and all numbers x > 1 are positive; therefore, the solution to $x^2 + 5x - 6$ ∃ 0 is x # -6 or x ∃ 1

5. Vertex: (3/2, 9/2)
Zeroes: {0, 3}

6. $$x = \frac{-7 \pm \sqrt{65}}{4}$$

7.



8. (d) 16 m by 16 m

9. (b) $(-\infty, -1] \cup \left[\frac{1}{2}, \infty\right)$

A2.14

1. (0,0) (0,4) (4,0) $\left(\frac{8}{3}, \frac{8}{3}\right)$

2. $z = 2\left(\frac{8}{3}\right) + 2\left(\frac{8}{3}\right) = \frac{32}{3}$ $\frac{32}{3}$ is the maximum value.

3. (b) 17 x + y # 6, x ∃ 1, x # 4 and y ∃ 0

The region bounded by these inequalities is represented to the left. Maximum and minimum values of 2x + 3y occur at the points of intersection.

(x,y)	3x + 2y
(1,0)	2
(4,2)	14
(1,5)	**17**
(4,0)	8

maximum values occurs at (4,2)

A2.15

1. (c) (-4, 0) χ (0, 4)

2. (d) c = Πd

3. 3 This problem illustrates an inverse variation xy = k where k is a constant.
$x_1y_1 = x_2y_2$ If $x_1 = 300$ and $y_1 = 4$ then $x_2 = 400$ and y_2 is unknown.

300 A 4 = 400 A y_2 $\frac{(300)(4)}{400} = y_2$ $3 = y_2$

4. $676

A2.16

1. (a) (-3, 2)

2. Answers will vary: $y = -(x - 1)(x + 1) = -x^2 + 1$

3. (5, 6)

4. (c) a hyperbola

$4x^2 + 6x - 5y2 + 2 = 0$ when put in standard form is

$4x^2 + 6x - 5y^2 = -2 \; \Psi \; 4(x^2 + 3/2x + 9/16) - 5y^2 = -2 + 9/4$

$4(x + 3/9)^2 - 5y^2 = 1/4$

$\frac{(x + 3/4)^2}{1/16} - \frac{y^2}{1/20} = 1$ is of the form of a hyperbola

$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$$

5. Center: (2, -3)
Radius = 5

A2.17

1. a. $|x - 1| > 2$
$x - 1 > 2$ or $x - 1 < -2$
$x > 3$ or $x < -1$

b. $|x - 1| < 2$
$-2 < x - 1 < 2$
$-1 < x < 3$

c. $|x - 1| = 2$

x - 1 = 2 or x - 1 = - 2
x = 3 or x = - 1

2.

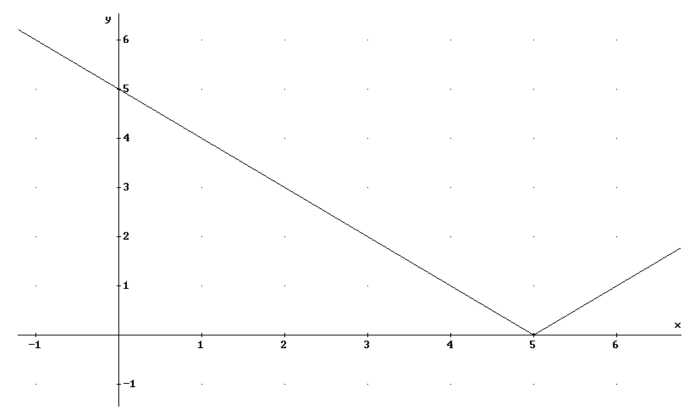
$x = \frac{16}{3}, \frac{-14}{3}$

3. $x \geq -\frac{2}{3}$ or $x \leq -\frac{10}{3}$

|3x + 6| ∃ 4 Ψ 3x + 6 ∃ 4 or 3x + 6 # - 4 Ψ 3x ∃ -2 x ∃ - 2/3 or
3x # -10 x # -10/3

4. (c) x = 3 and x = - 4

5.



6. (c) 6, -1

7. (d) 1, -5

8. (a) $\left(-\infty, \; -\frac{15}{2}\right) \cup \left(\frac{21}{2}, \; \infty\right)$

9. (b) $\left(-\infty, \quad -\frac{1}{2}\right) \cup \left(\frac{11}{2}, \quad \infty\right)$

10. (b) $\left[-2, \quad -\frac{4}{3}\right]$

11. (a) $\left[\frac{3}{5}, \quad \frac{9}{5}\right]$

A2.18

1. (c) $\pi^{y} = 2$

2. (d) 1.58

3. a. $\log_4 64 = 3$

   b. $2 \log_3 x + 5 \log_3 y$

4. (a) $2^y = x$ $y = \log_2 x$ then by the definition of logarithm $2^y = x$

5. (d) 61 $(x + 3)^{2/3} = 16$ raise both sides to the 3/2 power.

6. $x \approx 3.096$ Solution: $\log(3^{x-1}) = \log 10$ Ψ $(x - 1) \log 3 = \log 10$

A2.19

1. $v(x) = x(11 - 2x)(8.5 - 2x)$

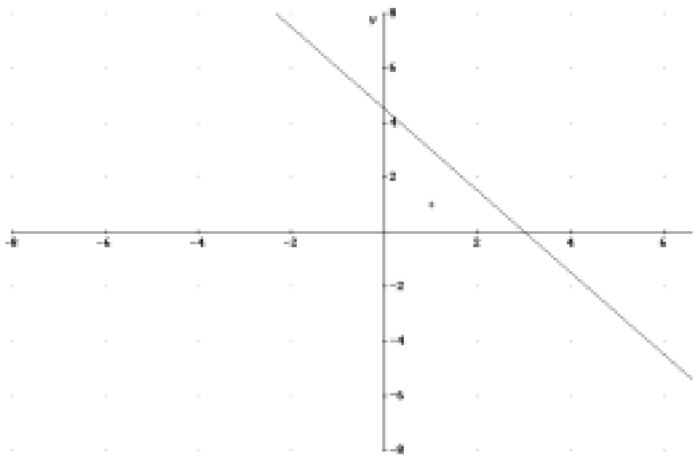
2. $v(x) = 5[(x)(11 - 2x)(8.5 - 2x)] + 4x^3$

A2.20

A2.21

A2.22

1.



2. (c) $\frac{11}{3}$

A2.23

1. (d) $(-\infty\ ,\ \frac{7}{5}]$

2. (b) (6, 4)

A2.24

1. Enter the function into your graphing calculator.

Use trace and zoom to approximate where the function crosses the x-axis. This happens approximately at values of 0.8 and -2.5

2.. a. (c) $\frac{3}{2}$

b. (c) - .4, .5

3. a. (a) (-4, 2) χ (4, 4)

b. No real solution

A2.25

1. $8x^3 > 125$

$$x^3 > \frac{125}{8}$$

$$\left(x^3\right)^{\frac{1}{3}} > \left(\frac{125}{8}\right)^{\frac{1}{3}}$$

$$x > \frac{5}{2}$$

2. a. (c) $\frac{3}{5}$

b. (d) 2

c. (c) $\left(-\infty,\ \frac{5}{2}\right)\cup\left(\frac{5}{2},\ \infty\right)$

d. (d) $\left(-\ \infty,\ \frac{1}{2}\right)\cup\left(\frac{1}{2},\ \infty\right)$